RIDDLES MATHS SE

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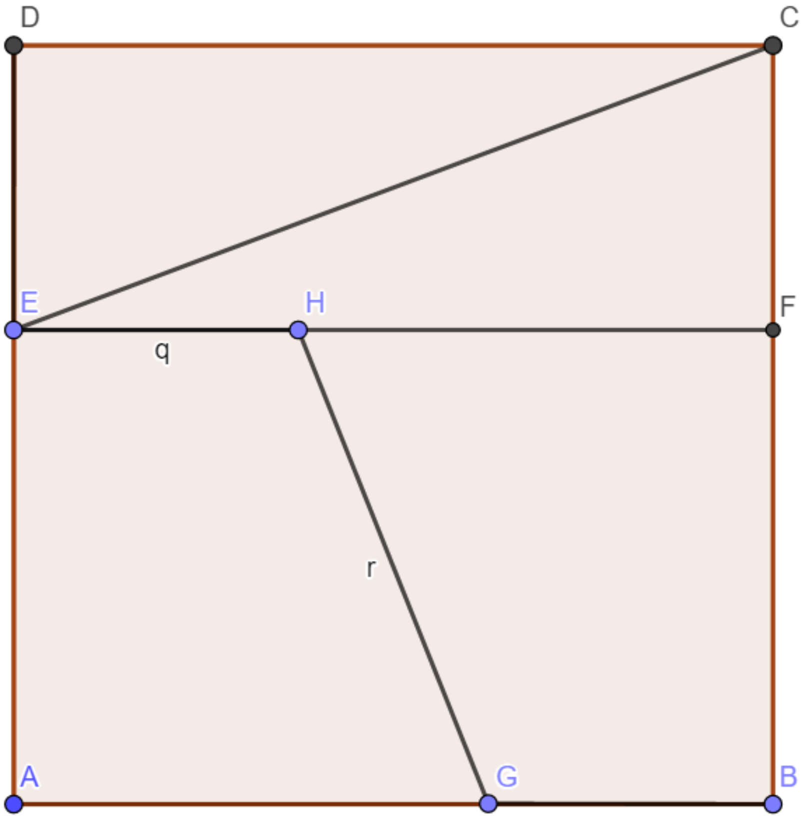
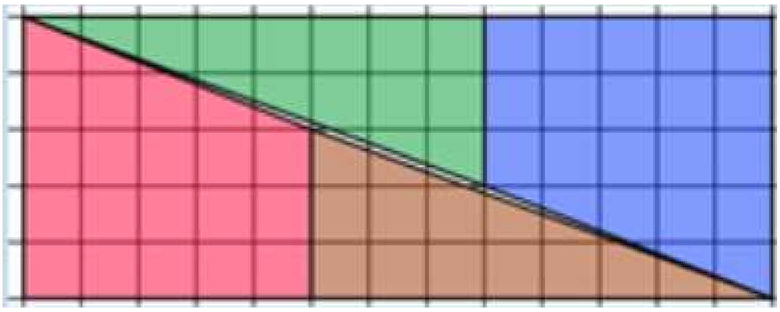
RIDDLE 1 -

Basically, this problem is telling us that it is possible to give birth to a square in a 8x8 bigger square just by moving pieces of it, which, obviously, is absurd. That's why we are going to prove that it's mathematically impossible.
First, let's have a look at this figure and manipulate it on GeoGebra :



Here is the 8x8 cut down into four pieces as shown in the paper.
Now, let's rearrange the pieces:

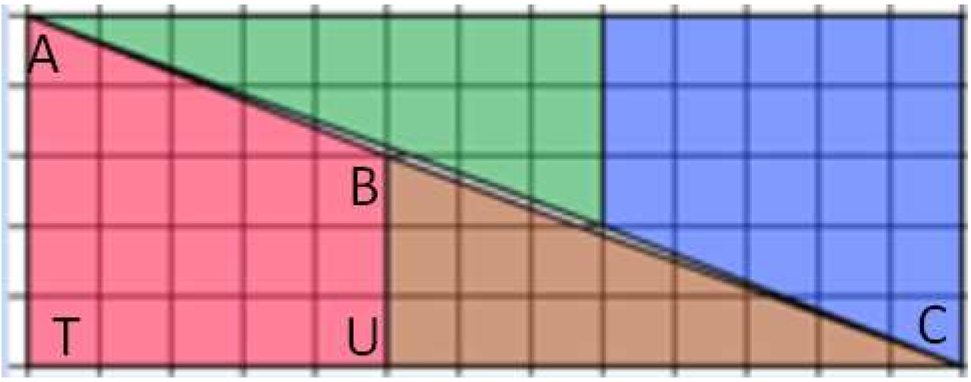


(To see more easily, we added colors)

Now, we can clearly see that there is a gap between the two big triangles which reveals a little problem… Indeed, if the pieces aren't attached, the problem would make no sense ! If this theory is true, it would mean that the new-born square only exists thanks to the gap.

Let's now try to prove mathematically everything we already conjectured:

To recap, we originally have a 8x8 square, constituted of 64 squares, and the rearranged figure is a 13x5 rectangle which now contains 65 squares. Obviously, and only if we really rearranged the pieces correctly, which includes the fact that they are attached together, the two areas have to be equal, which is not the case here. We have seen that there may be a gap between the two triangles in the rectangle, let's try to prove its existence by seeing if the points are aligned. Let points A, B, C, U and T :



Let's try to use a reductio ad absurdum and the contraposition of Thales' theorem.

Assuming that points A, B and C are aligned, and that segments [AT] and [BU] are parallel, we can use the contraposition Thales' theorem :

CB/CA = CU/CT = BU/AT

With AT = 5, BU = 3, CU = 8, CT = 13, we have :

3/5 = 8/13 which is completely absurd (13*3 = 39 ≠ 8*5 = 40)

This all means that the points A, B and C are not aligned because the condition required by the theorem is not confirmed here, that's what we call the contra-posed !

(This also works with the green and blue pieces, knowing that they have the same areas as the brown and red ones)

Finally, all this was an optical illusion and, in reality, the red and brown pieces don't form a triangle, which explains everything : this is not a real rectangle and there is well and truly a gap in it, that's where the "new-born square" was really born !

(All this problem would have been solved in less than five minutes if we could have seen the gap in the photocopy that teachers gave us, but they filled it !)

RIDDLE 2 -

For this Riddle, we are facing a probability problem, which we clearly know how to solve.

First of all, from the information we have in the statement of this problem, we know that initially, the color of the first token was decided by the flip of the coin and that therefore given that the second token is necessarily white then the cases of the content of the bag are either that the token is white or black, so that the bag has a black and white token or two white ones, both cases are equally probable

So let's make a table to visualize the problem more easily :

First token	Second one (white)	Probability
White	White	1/2
Black	White	1/2

Therefore let's see more precisely the four equally probable outcomes of pulling one token from the bag by not counting the drawing.

Case	Probability	Remaining token's color	Outcome color
White & White	1/2	White	White
--------------------------		White	White
Black & White	1/2	White	Black
--------------------------		Black	White

So, we can see that the chance for the left token color to be white is ¾. Nevertheless, we want to be sure a hundred percent that the outcome will give a white token, which now depends on the first token's color. Then, we have to exclude the event in red that gives a black token as an outcome. Which leaves us with three equal probable events : either it's white, black or white. Finally, the chance of drawing a white counter is ⅔.

RIDDLE 3-

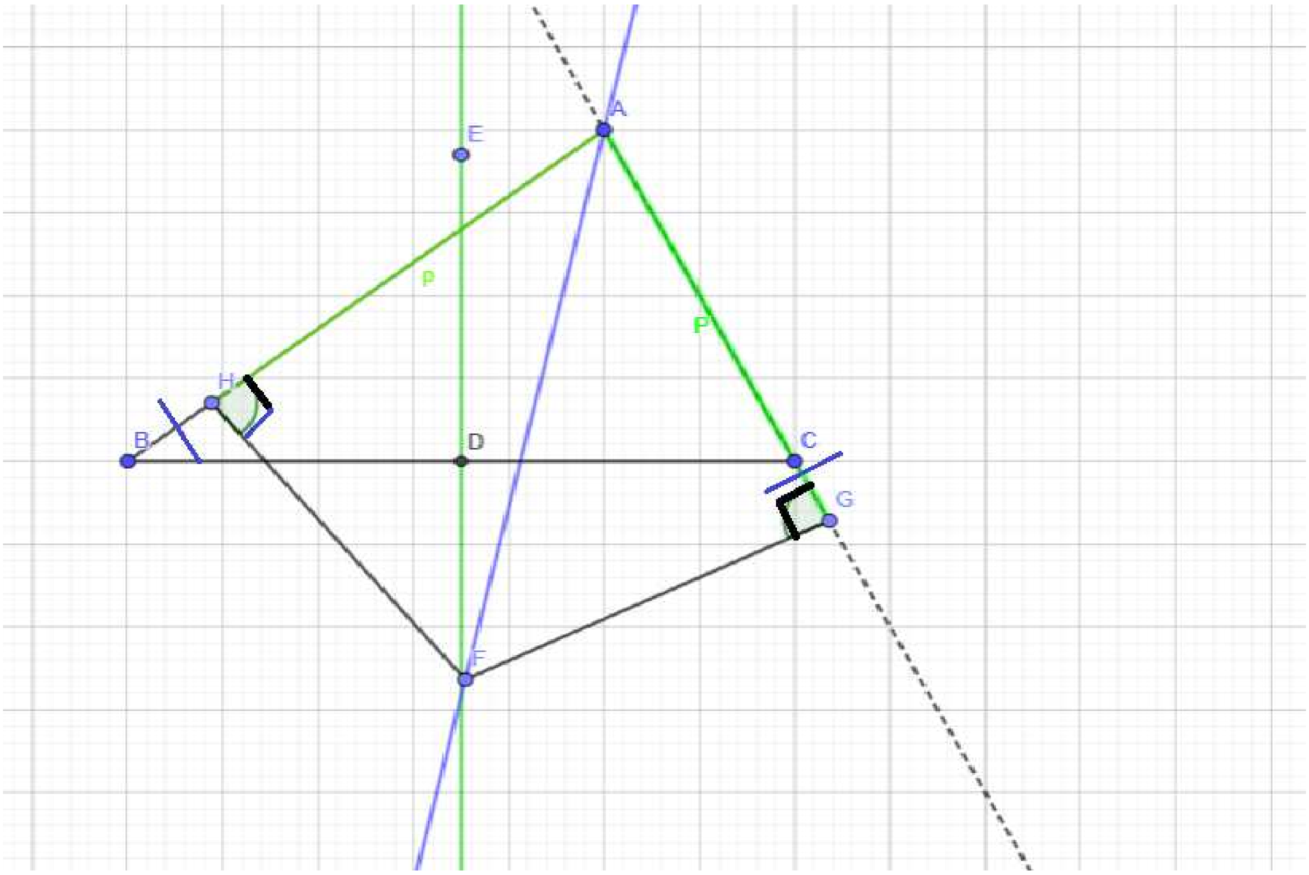
In order to find the fallacy present in the given proof from Lewis Carrol regarding the fact that **ALL TRIANGLES ARE ISOSCELES**, we are going to take in account the proof's elements

In triangle ABC, we can figure out that line (DE) seems to be bisector of triangle FCB, moreover according to the representation of the triangle so according to the **Angle Bisector Theorem** FB=FC. Therefore we notice that the line (DE) is common to triangle FBH and FCG, let's figure out that these triangles are in fact equal due to the **Theorem of Congruence of Triangles.** Indeed, let's figure out that the triangles are equal because :
-There is two right angle at H and G
-FH=FG
-FB=FC
So according to these facts HB=GC

It also seems that line (AF) is common to two triangles AFH and AGF but does this really prove that these two triangles are equal ? Let's try to figure out if these two triangles are in fact equal according to the **Theorem of Congruence of Triangles.** Indeed, let's figure out that these triangles are the same because :
-the two angles at A are equal

- According to the **Angle Bisector Theorem** ⇒ FH=FG

So according to all the facts we mentioned in this paragraph HA=GA
Now in order to verify that ABC is isosceles, let's draw (Geogebra) the triangle according to the datas we mentioned previously :



So as this construction shows us, according to the datas which proves that ABC is isosceles and after applying them, we notice that point F will be out of ABC, which concludes the fact that ABC isn't isosceles. Moreover, if ABC had been isosceles, the two bisectors AE and AF would have had to be superimposed in some way.

Here is an inspirational quote from him to introduce our article : "*The best way to achieve the impossible is to believe that it is possible.*"

Who does not know **Alice in Wonderland**, a famous novel who has rocked children since its publication. This famous novel is due to a British writer, **Lewis Caroll**. However, despite the fact that the latter was able to mark the minds of generations of children through his literature, it should be known that he was an eminent scientist but above all an eminent mathematician who since his youngest age has excelled in this field...

Who he really was...

While he is known today primarily as an author, Lewis Carroll, or rather Charles Dodgson, was also a brilliant scholar. He taught math and logic to undergraduates at Christ Church, Oxford, for many years, at the same time he was creating fanciful stories to entertain children. He carved a unique place for himself in his field when he combined his two areas of expertise, writing entertaining scenarios for challenging math and logic problems, or turning the concepts into games, to make the learning easier, more enjoyable, and more lasting. (This method was very modern in his time because mathematics education was purely theoretical and "strict") While he did not produce any revelatory findings, many of his ideas and theories are still respected today, and his creative, user-friendly approach to learning remains fresh today. Current mathematical scholars around the globe are discovering a new appreciation for his contributions to the field.

His career

He was born on January 27, 1832 in Daresbury (a small village near Manchester). His father is a priest of the Anglican Church (a branch of the Church that does not recognize the Pope). In 1856, he devoted himself to the nascent art of photography, now under the pseudonym of Lewis Carroll, which he invented by translating his first names into Latin, reversing the order of the letters, then translating them again into english. He published his first photos in "The Train" magazine: pictures of Alice and her two sisters in the Liddells' garden. Her favorite subject remains the little girls whom Alice Liddell met at that time. She will inspire him to write the adventures of Alice in Wonderland. The process of creating the book begins when the young professor receives permission from Mrs. Liddell, wife of the college dean and mother of Alice, to date her three daughters and use them as models for his photographs. Around 1850, he began to photograph young girls in poses of fairy-tale heroines, then moved to undressed shots. "I hope you will allow me to at least photograph Janet nude; it seems absurd to have the slightest qualm about the nudity of a child of this age. By this quote, we can understand the accusations of pedophilia against Lewis Carroll. It was in 1862, during a boat trip, that Lewis Carroll began to tell young girls a wonderful tale, of which he named the main character Alice, in reference to Alice Liddell. It will take three years to finalize his work. The work is still a universal success today. At the same time fundamental in children's literature, it also triggers passions in adults, in particular for logic games and word games that it develops.

His mathematician life in 7 dates

→ December 1854, he brilliantly graduated in mathematics, which he taught with logic at a girls' school in Oxford. However, he does not like teaching.

→ In 1860, he published elements on algebraic plane geometry.

→ In 1879, he published “Euclid and His Modern Rivals”

→ In 1881, he abandoned photography and devoted himself to logic.

→ In 1886, he published “The Game Of Logic”

→ In 1896, he created the concept of “Symbolic Logic”

→ He continued his career and died on January 14, 1898 at the age of 66 in Guildford.

His place in the world of mathematics...

Lewis Caroll somewhat revolutionized the way of interacting with mathematics in his day. Indeed, the mathematician writer mixed the fields of writing and mathematics in order to allow people to learn mathematics in a more fun, more attractive way. As we specified previously he created scenarios for challenging math and logic problems, or turning the concepts into games.
Moreover, In mathematics, Dodgson worked primarily in the areas of geometry, linear algebra, matrix algebra, mathematical logic, and recreational mathematics, producing nearly a dozen books under his real name. Dodgson also developed new ideas in linear algebra (for example, the first test of the Rouché-Fontené theorem), in probability, and in the study of elections (for example, Dodgson's method) and committees; some of this work was not published until well after his death. His occupation as a math teacher at Christ Church gave him some financial security.
His mathematical work attracted renewed interest at the end of the twentieth century. Martin Gardner's book on Logic Machines and Diagrams and William Warren Bartley's posthumous publication of the second part of the book on Carroll's Symbolic Logic sparked a reassessment of Carroll's contributions to symbolic logic. Robbins and Rumsey's studies of Dodgson's condensation, a method of evaluating determinants, led them to the alternating sign matrix conjecture, now a theorem. The discovery in the 1990s of additional ciphers Carroll had constructed, in addition to his "Memoria Technica," showed that he employed sophisticated mathematical ideas in their creation.

What he brought to math...

In the last decades of his life, Carroll published three mathematical pieces in *Nature*. The first, on a method for finding the day of the week for any date (L. Carroll *Nature* 35, 517; 1887), reflects the calendar problems of the time: to obtain information on future days and dates, you had to consult an almanac. Carroll found mental calculation methods gripping. Introducing the piece, he wrote, “I am not a rapid computer myself”, yet noted that he could do ten such problems in less than four minutes. His rule uses four integer calculations: two for the year, the third for the month and the last for the day.
Carroll did not influence his contemporary colleagues in the development of mathematical ideas. However, posthumously, beginning in the last half of the twentieth century, his contributions to voting theory were uncovered in three papers written between 1874 and 1876. The third, 'A Method of Taking Votes on More Than Two Issues', is the most important. Carroll was the first to create a voting method that would achieve biproportional representation — that is, proportionality with respect both to the population in the districts and to the apportionment of seats to the political parties in the legislature. Despite Carroll's friendship with Lord Salisbury, the UK prime minister at the time, it was not applied for political reasons. (Today, the European Parliament uses a form of proportional representation.)

His mathematical works

- *A Syllabus of Plane Algebraic Geometry*) (1860)
- *Notes on the First Two Books of Euclid, Designed for Candidates for Responsions*(1860)
- *Condensation of Determinants, Being a New and Brief Method for Computing their Arithmetic Values* (1866)
- *The Fifth Book of Euclid Treated Algebraically* (1858 and 1868)
- *An Elementary Treatise on Determinants with their Application to Simultaneous Linear Equations and Algebraic Geometry* (1867)
- *The Enunciations of Euclid I-VI, Together with Questions on the Definitions, Postulates, Axioms, &c.* (1873)
- *Euclid and His Modern Rivals* (1879)
- *The Alphabet Cipher* (1868)
- *Euclid, Books I, II* (1882)
- *Supplement to Euclid and His Modern Rivals* (1885)
- *The Game of Logic* (1887)
- *Curiosa Mathematica I* (1888)
- *Curiosa Mathematica II* (1892)
- *Symbolic Logic Part I*
- *Symbolic Logic Part II*
- *The Theory of Committees and Elections* (1958)

His most famous riddles (apart from those you proposed us)

1. You are given two glasses. One contains 50 tablespoons of milk, the other 50 tablespoons of water. Take one tablespoon of milk and mix it with the water. Now take one tablespoon of the water/milk mixture and mix it with the pure milk to obtain a milk/water mixture. Is there more water in the milk/water mixture or more milk in the water/milk mixture?
2. If you paint the faces of a cube with six different colors, how many ways are there to do this if each face is painted a different color and two colorings of the cube are considered equivalent if you can rotate one to get the other? What if we drop the restriction that the faces be painted different colors?
3. Make a word-ladder from FOUR to FIVE. (Every step in a word ladder differs from the previous step in exactly one letter and each step in the ladder is an English word.)
4. Why is a raven like a writing desk?

The hidden math behind "*Alice In Wonderland*"

What would Lewis Carroll's *Alice's Adventures in Wonderland* be without the Cheshire Cat, the trial, the Duchess's baby or the Mad Hatter's tea party? Look at the original story that the author told Alice Liddell and her two sisters one day during a boat trip near Oxford, though, and you'll find that these famous characters and scenes are missing from the text. However, can mathematics also be used as a creative stimulus? As radical as this thought might be, the works of Charles Lutwidge Dodgson, more famously known as Lewis Carroll, indicate that it is not beyond the realm of possibility.

As we saw earlier, Caroll's philosophy regarding mathematics consisted of allowing every kind of person to understand "advanced mathematics ''. Therefore Lewis Caroll took his mathematics to his fiction. Using a famous mathematics technique, reductio ad absurdum. Indeed he picked apart the "semi logic" of the "abstract" mathematics and tends to logical conclusions, with incredible results. The outcome of this process is **Alice's Adventures in Wonderland** !

In fact, through reading this novel, we notice many mathematicals references in some scenes. First, one of the scene, where mathematical subtleties are the most present, is **the encounter between Alice and the Caterpillar who was smoking a hookah pipe**.While some have argued that this scene, with its hookah and "magic mushroom", is about drugs, It seems that it's about what Dodgson saw as the absurdity of symbolic algebra. Indeed, for example the word "hookah" is of arabic origin such as the word "algebra". Moreover, Augustus De Morgan, the first British mathematician to lay out a consistent set of rules for symbolic algebra, uses the original Arabic translation in "*Trigonometry and double algebra*". He calls it "al jabr we al mokabala" or "restoration and reduction" which almost exactly describes Alice's experience when she fell into it, he directly led to her meeting with the Caterpillar . Restoration was what brought Alice to the mushroom: she was looking for something to eat or drink to grow to her right size again, and reduction was what actually happened when she ate some: she shrank so rapidly that her chin hit her foot.

On the other hand, in this novel, Lewis Caroll cannot stop making riddles !! Decidedly, the British mathematician is passionate about logical riddles and passes his passion through his literature. Indeed, we can observe this in the scene "**At the tea party**" in which we can find the following extracts:

-“Take some more tea,” the March Hare said to Alice, very earnestly. “I’ve had nothing yet,” Alice replied in an offended tone, “so I can’t take more.”

-“You mean, you can’t take less,” said the Hatter: “it’s very easy to take more than nothing.”

Here, The Mad Hatter is trying to tell Alice that she can have more tea, given that she has not yet had anything to drink, but what she cannot do is “take less.”.